HIGH SCHOOL

EST II
MATH – LEVEL 2

Date:

Test Center:

Room Number:

Student's Name:

National ID:

EST ID:

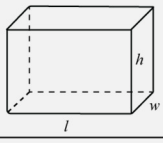
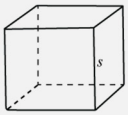
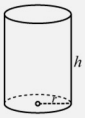
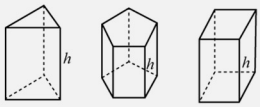
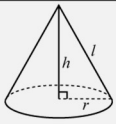
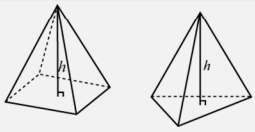
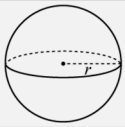
Duration: 60 minutes

40 Multiple Choice Questions

Instructions:

- Place your answers on the answer sheet. Mark only one answer for each of the multiple-choice questions.
- Graphing and scientific calculators are allowed.
- The formula sheet is available on the following page of the booklet for your reference.

Formula Sheet

	Volume	Surface area	Lateral surface area
Cuboid 	$V = lwh$	$S.A. = 2(lw + lh + wh)$	
Cube 	$V = s^3$	$S.A. = 6s^2$	
Cylinder 	$V = \pi r^2 h$	$S.A. = 2\pi rh + 2\pi r^2$	
Prism 	$V = Bh$ (where B is the area of one of the bases, whether it is a square, a circle, a triangle, or a polygon)		
Cone 	$V = \frac{1}{3}\pi r^2 h$	$S.A. = \pi rl + \pi r^2$	$L.A. = \pi rl$
Pyramid 	$V = \frac{1}{3}Bh$ (where B is the area of the base)		
Sphere 	$V = \frac{4}{3}\pi r^3$	$S.A. = 4\pi r^2$	
Hemisphere 	$V = \frac{2}{3}\pi r^3$	$S.A. = 3\pi r^2$	$L.A. = 2\pi r^2$

Question 1. Let M be the point with affix z such that $|z + 3i| = |z - i|$. The point M moves on:

A. A circle with center A
B. A circle with center $(1, 2)$
C. The line with equation $x = -1$
D. The line with equation $y = -1$
E. The line with equation $y = 2$

Question 2. Given two functions f and g defined on $\mathbb{R}$ by $f(x) = (x - p)^2 + (x + q)^2 - 4$ and $g(x) = (p + q)x^2 - 4x + k - 3p + 2q$. If $f(x) = g(x)$ for all values of x, then $k =$

A. 2
B. 4
C. 5
D. 6
E. 8

Question 3. Consider the following functions:
$f_1(x) = -x^2 - c_1x + k$, $f_2(x) = (x - 1)^2 - 4x + c_2$, and $f_3(x) = x^2 - 2x + c_3$.
The axis of symmetry of f_1 is $x = 1$; the product of roots of f_2 is equal to their sum; and f_3 has exactly one double root. What is the value of $c_1 + c_2 + c_3$?

A. -4
B. 1
C. 4
D. 6
E. 9

Question 4. Which of the following are the solution(s) of $\cos(2x) + 1 = \sin(2x)$ in the interval $\left[0, \frac{\pi}{2}\right]$?

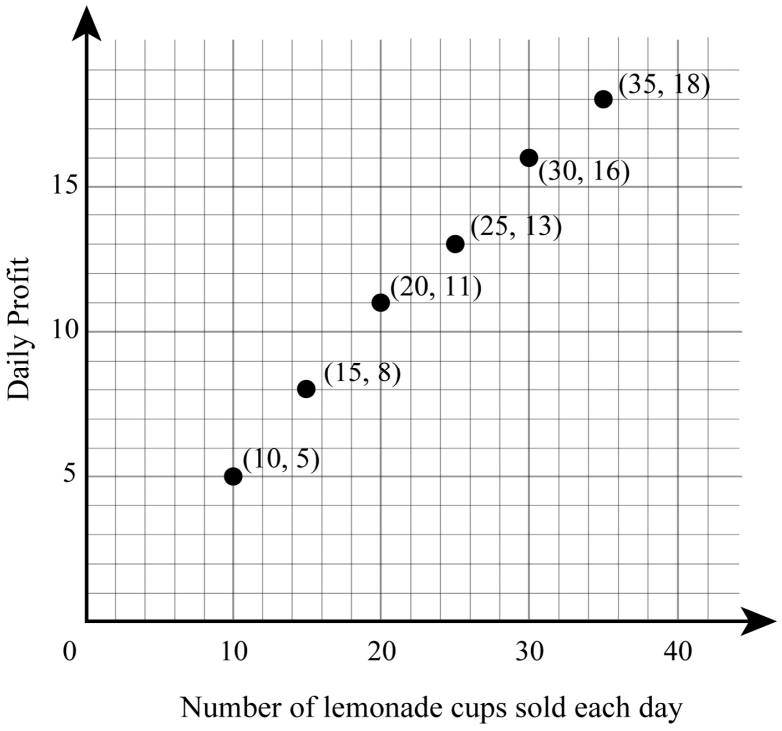
A. $\frac{\pi}{6}$ and $\frac{\pi}{2}$

B. $\frac{\pi}{2}$ and $\frac{3\pi}{2}$

C. $\frac{\pi}{4}$, $\frac{\pi}{2}$, and $\frac{3\pi}{2}$

D. $\frac{\pi}{4}$ and $\frac{\pi}{2}$

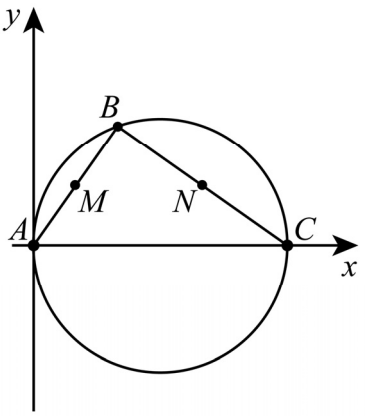
E. None of the given options is correct.



Question 5. Karim runs a lemonade stand during the summer. He keeps track of lemonade cups sold each day and the corresponding daily profit. After a week, starting from Monday, the data is given by the graph above. Using the line of best fit, what is Karim's profit, to the nearest dollar, if he sells 40 cups of lemonade on Sunday?

A. \$19
B. \$20
C. \$21
D. \$22
E. \$25

Question 6. Given a circle R with equation $(x-3)^2+y^2=9$, as shown in the adjacent figure, points A, B, and C are drawn on R with $\overline{AC}$ being the diameter. If M and N are the respective midpoints of the chords $\overline{AB}$ and $\overline{CB}$, what is the value of MN? *(Figure not drawn to scale)*



A. $MN=\sqrt{10}$ units
B. $MN=3$ units
C. $MN=\sqrt{5}$ units
D. $MN=\frac{3\sqrt{2}}{2}$ units
E. $MN=\frac{2\sqrt{3}}{3}$ units

Question 7. If $f(x) = 3x^2 - 5x - 2$ and $g(x) = x^2 + 4x - 6$, the solution of the inequality $f(x) > g(x)$ is:

A. $x < \frac{1}{2}$ or $x > 4$

B. $x \leq \frac{1}{2}$ and $x \geq 4$

C. $x > \frac{1}{2}$ and $x < 4$

D. $-2 < x < 4$

E. $\frac{1}{2} \leq x \leq 4$

Questions 8 and 9 refer to the information below:

Function g is defined on $\mathbb{R}$ by $g(x) = \frac{2e^x}{e^x+1} - 1$.

Question 8. The inverse function of g is g^{-1}. The domain of definition of g^{-1} is:

A. $(-\infty, +\infty)$

B. $(-1, 0)$

C. $(-\infty, 1)$

D. $\left(e, \frac{1}{e}\right)$

E. $(-1, +1)$

Question 9. The explicit expression of $g^{-1}(x)$ is:

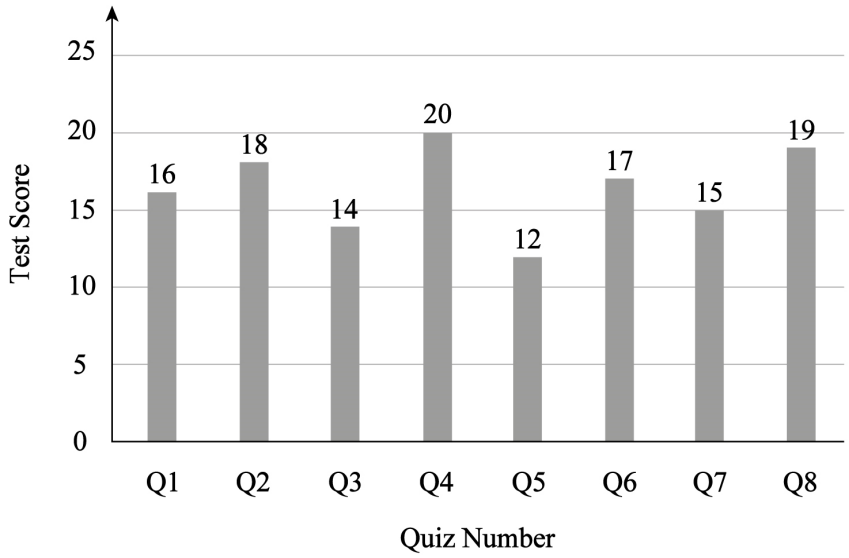
A. $g^{-1}(x) = \ln\left(\frac{1+x}{1-x}\right)$

B. $g^{-1}(x) = \ln\left|\frac{1-x}{1+x}\right|$

C. $g^{-1}(x) = \frac{e^x+1}{2e^x} + 1$

D. $g^{-1}(x) = \ln\left(\frac{2e^x}{e^x+1} - 1\right)$

E. $g^{-1}(x) = \ln\left(\frac{2e^x}{e^x+1} + 1\right)$



Question 10. The chart above displays Hala's scores (out of 20) on her math quizzes already completed. Hala wants to take another test so that her new average is more than her present one by 0.125 points. What should be her score on the new quiz?

A. 16.25
B. 17.50
C. 18.75
D. 19.75
E. 20.0

Question 11. The two matrices A and $B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ verify $A - 2B = \begin{pmatrix} 1 & 3 \\ -1 & 1 \end{pmatrix}$. Find A.

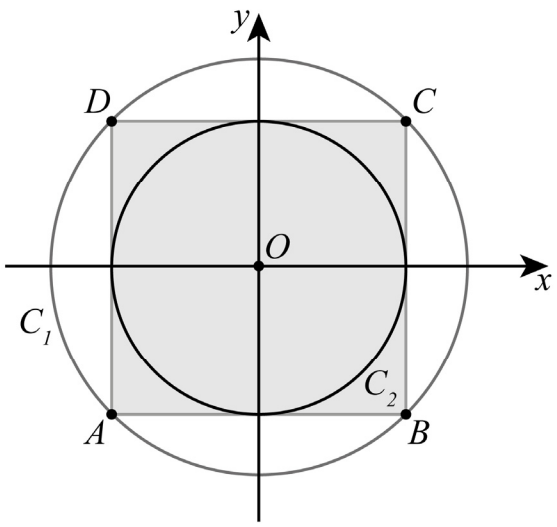
A. $\begin{pmatrix} 3 & -7 \\ 5 & 9 \end{pmatrix}$

B. $\begin{pmatrix} 3 & 7 \\ 5 & 3 \end{pmatrix}$

C. $\begin{pmatrix} 3 & 7 \\ 5 & 9 \end{pmatrix}$

D. $\begin{pmatrix} 3 & 5 \\ 7 & 9 \end{pmatrix}$

E. $\begin{pmatrix} 5 & 7 \\ 5 & 9 \end{pmatrix}$



Question 12. In the figure above, C_1 is a circle with an area of 25 square units, circumscribed about a square $ABCD$ with center at the origin O. Circle C_2 is inscribed inside the square. What is the equation of the small circle? *(Figure not drawn to scale)*

A. $x^2 + y^2 = 25\pi$

B. $x^2 + y^2 = 12.5\pi$

C. $x^2 + y^2 = 5\pi$

D. $x^2 + y^2 = 4\pi$

E. $x^2 + y^2 = \frac{25}{2\pi}$

Question 13. Let $A = (2, 0)$ and $B = (1, 1)$. The graph of the function $h(x) = 2^{x-a}$ passes through point C, the midpoint of $\overline{AB}$. What is the value of a?

A. $\frac{5}{2}$

B. 2

C. $\frac{3}{2}$

D. 1

E. -1

Question 14. Given the polynomial $P(x) = x^5 + 2x^3 + Ax + B$, where A and B are two constants. When $P(x)$ is divided by $x - 1$, the remainder is 2, and when $P(x)$ is divided by x, the remainder is -5. What is the value of A?

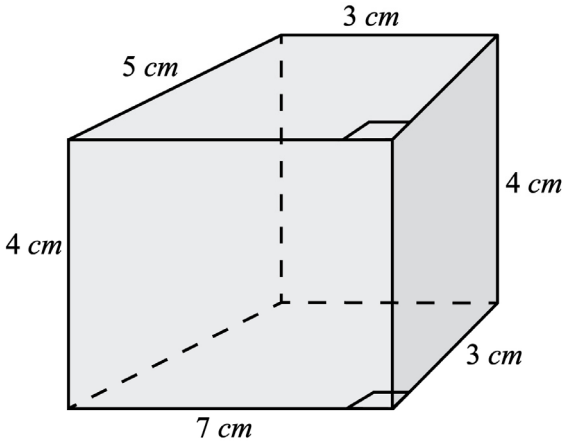
A. $A = -5$
B. $A = 0$
C. $A = 2$
D. $A = 4$
E. $A = 5$

Question 15. A factory produces three types of products: A, B, and C, in the ratio of $5:3:2$, respectively. The probabilities that a factory product is defective are: 0.10 for product A, 0.15 for product B, and 0.25 for product C. To the nearest thousandth, what is the probability that a randomly selected product is of type B, given that it is not defective?

A. 0.255
B. 0.298
C. 0.315
D. 0.350
E. 0.425

Question 16. Point M has the coordinates $\left(2\cos\left(\frac{\pi}{6}\right), 2\sin\left(\frac{\pi}{6}\right)\right)$. The equation of the line passing through M and the origin is:

A. $y = \sqrt{3}x$
B. $y = x$
C. $y = \frac{1}{\sqrt{3}}x$
D. $y = \frac{1}{3}x$
E. $y = -\sqrt{3}x$



Question 17. A block of wood is a right prism and has the dimensions shown in the diagram above. What is the surface area of the block of wood? *(Figure not drawn to scale)*

A. $96\ cm^2$
B. $102\ cm^2$
C. $111\ cm^2$
D. $182\ cm^2$
E. $185\ cm^2$

Question 18. Each evening, Jamil eats $\frac{4}{10}$ of the pistachios left in his jar. After two evenings, 27 pistachios remain. How many pistachios were initially in the jar?

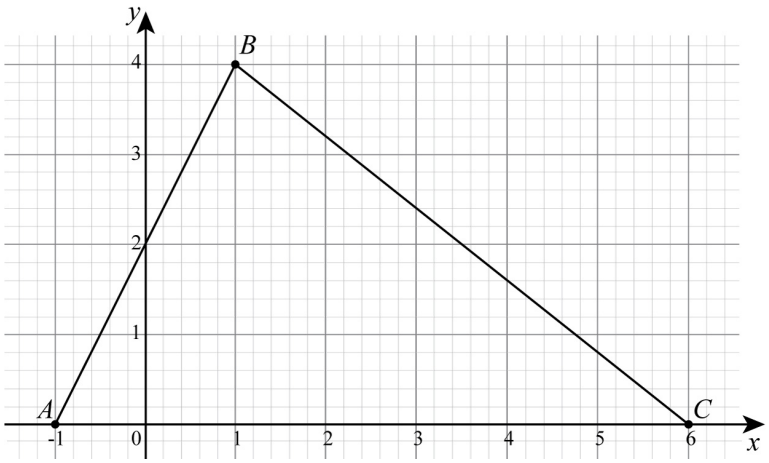
A. 12
B. 34
C. 50
D. 75
E. 120

Question 19. The value of $\lim_{x\to 1}\left(\frac{\sqrt{x+3}-1}{x-1}\right)$ is:

A. -2
B. $\frac{1}{4}$
C. $\frac{2}{3}$
D. 1
E. ∞

Question 20. The fourth term and the seventh term of an arithmetic series are 23 and 41, respectively. The first term of the sequence is:

A. 32
B. 23
C. 11
D. 6
E. 5

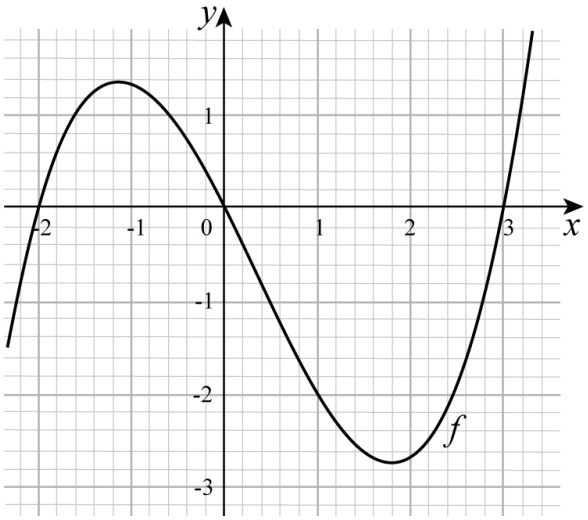


Question 21. The approximate measure of the angle $\angle BAC$, rounded to the nearest degree, is:

A. 88°
B. 72°
C. 63°
D. 56°
E. 45°

Question 22. Given the points $A(1,2,1)$, $B(2,2,1)$, $C(-2,1,3)$, and $D(3,2,5)$, the sum of all edges' lengths of tetrahedron $ABCD$ is approximately:

A. 15.5 units
B. 20.0 units
C. 23.2 units
D. 23.4 units
E. 24.0 units



Question 23. The graph above represents the function f defined by $f(x) = ax^3 - \frac{1}{3}x^2 + bx + c$ where a, b, and c are three real numbers. Find the values of a, b, and c.

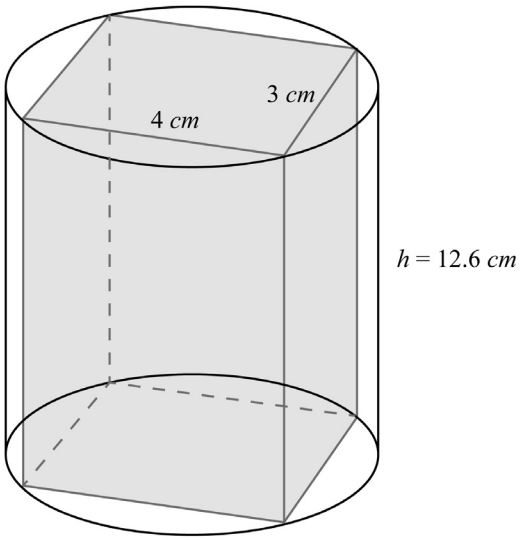
A. $a = 1, b = -1, c = 2$

B. $a = \frac{1}{3}, b = -2, c = 0$

C. $a = -\frac{1}{3}, b = -1, c = -2$

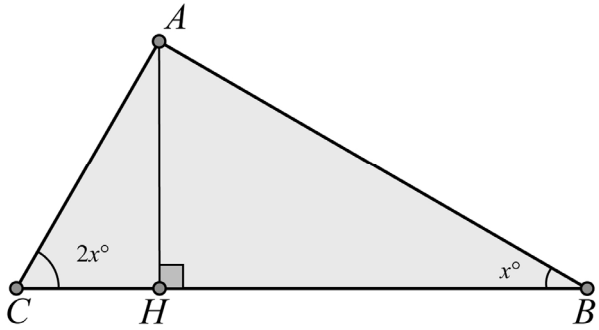
D. $a = -\frac{1}{3}, b = \frac{1}{3}, c = 0$

E. $a = -2, b = 1, c = 0$



Question 24. A rectangular prism is inscribed in a cylinder, as shown in the figure above. The volume of the space between the prism and the cylinder is: *(Figure not drawn to scale)*

A. $70.2\ cm^3$
B. $96.2\ cm^3$
C. $107.9\ cm^3$
D. $147.3\ cm^3$
E. $151.2\ cm^3$



Question 25. In the figure above, ABC is an acute triangle such that $BC = 4\sqrt{3}\ cm$, $BH = \alpha\ cm$, $AH = 3\ cm$, $m\angle ABC = x°$, and $m\angle ACB = 2x°$. What is the value of x? (Figure not drawn to scale)

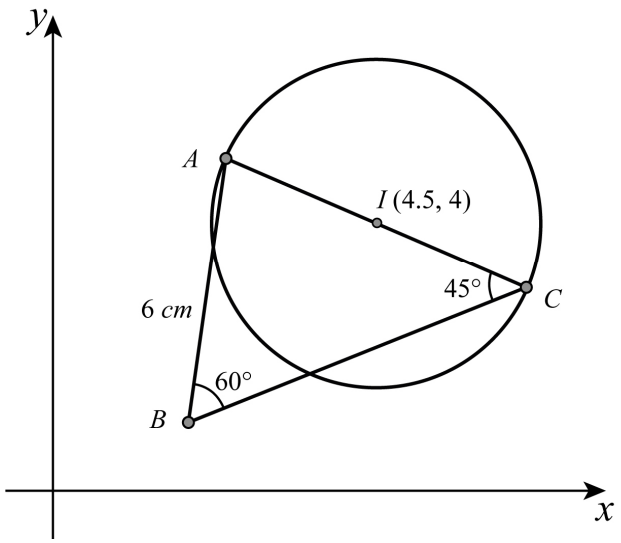
A. $x = 60$
B. $x = 50$
C. $x = 45$
D. $x = 30$
E. $x = 15$

Question 26. The simplified form of the expression $\frac{\sin(x+y)-\sin(x-y)}{\cos(x+y)+\cos(x-y)}$ is:

A. $\cot(y)$
B. $\csc(x)$
C. $\csc(y)$
D. $\tan(x)$
E. $\tan(y)$

Question 27. The equation $(x-1)e^{x^2-1}+1=x$ has:

A. 1 solution
B. 2 solutions
C. 3 solutions
D. 4 solutions
E. No solutions



Question 28. Triangle ABC has dimensions and angles as shown in the figure above. R is a circle with center I. The equation of the circle R is: *(Figure not drawn to scale)*

A. $(x-4.5)^2+(y-4)^2=6.0$
B. $(x-4.5)^2+(y-4)^2=8.5$
C. $(x-4.5)^2+(y-4)^2=9.66$
D. $(x-4.5)^2+(y-4)^2=13.5$
E. $(x-4.5)^2+(y-4)^2=17.0$

Question 29. For the points $A(2, 3, 2t + 1)$, $B(2t - 4, -3, 7)$, and $I(1, 2, 3)$, where t is a real number, let M be the midpoint of $\overline{AB}$ such that $MI = 3$. What is the largest possible value of t?

A. -1
B. 0
C. 1
D. 2
E. 3

Question 30. The function f with $f(x) = \cos(4x)$ has a period equal to:

A. $\frac{\pi}{4}$
B. $\frac{\pi}{2}$
C. $\frac{2\pi}{3}$
D. $\frac{3\pi}{2}$
E. 2π

Question 31. The product of vectors $v = (a, b) \in \mathbb{R}^2$ and $u = (c, d) \in \mathbb{R}^2$ is the vector $v \otimes u$ in $\mathbb{R}^4$ defined by (ac, ad, bc, bd). What is $(2, -3) \otimes (13, 4)$?

A. $(-36, -12, 26, 8)$
B. $(2, -3, 13, 4)$
C. $(26, 8, -39, -12)$
D. $(26, 8, -39, 12)$
E. $(26, 8, 8, 26)$

Question 32. Consider the word ARRANGE. How many distinct arrangements of this word can be formed such that no two A's are adjacent?

A. 1,620
B. 900
C. 720
D. 700
E. 360

x_i	6	7	8	9	10	11	12	13	14
Frequency	2	2	1	1	0	1	1	0	2

Question 33. The interquartile range of the data set given in the table above is:

A. 1.5
B. 2.0
C. 3.5
D. 4.0
E. 5.0

Question 34. A ball is shot into the air. Its height above the ground, t seconds after being shot, is given by the function $h(t) = -4t^2 + 16t + 2$ in meters. What is the maximum height h reached by the ball?

A. $h = 18.0\ m$
B. $h = 15.0\ m$
C. $h = 9.0\ m$
D. $h = 4.5\ m$
E. $h = 2.0\ m$

x	$f(x)$	$g(x)$
-3	-2	-4.5
-1	-1	-2.5
3	1	1.5
6	2.5	4.5
a	b	b

Question 35. In the table above, f and g are two real-valued linear functions. Which of the following represents the coordinates (a, b)?

A. $(-1, 2)$
B. $(1, 0.5)$
C. $(2, 0.5)$
D. $(2, 1.5)$
E. $(4, 2.5)$

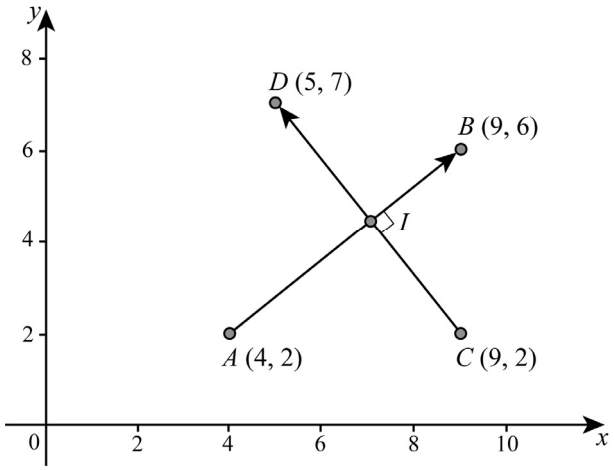
Question 36. Approximately, what is the value of the expression

$K = 2\sin\left(\frac{\pi}{9}\right) \times \cos\left(\frac{\pi}{9}\right) \times \cos\left(\frac{2\pi}{9}\right) \times \cos\left(\frac{3\pi}{9}\right) \times \cos\left(\frac{4\pi}{9}\right)$?

A. -0.043
B. 0.043
C. 0.140
D. 0.642
E. 1.043

Question 37. Given the parabola defined by the equation $y + x^2 = 6x - 5$, what is the area of the triangle formed by the vertex and the two x-intercepts?

A. 12 square units
B. 10 square units
C. 9 square units
D. 8 square units
E. 6 square units



Question 38. In the figure above, the value of $\left(\overrightarrow{IA}\right)^2 + \left(\overrightarrow{ID}\right)^2 + \left(\overrightarrow{IC}\right)^2 + \left(\overrightarrow{IB}\right)^2$ is equal to:

A. 16
B. 26
C. 42
D. 55
E. 75

Question 39. The inverse of the matrix $M = \begin{pmatrix} -1 & 1 \\ -2 & 2 \end{pmatrix}$ is:

A. $M^{-1} = \begin{pmatrix} 1 & -1 \\ 2 & 2 \end{pmatrix}$

B. $M^{-1} = \begin{pmatrix} 0 & -1 \\ 2 & 0 \end{pmatrix}$

C. $M^{-1} = \begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix}$

D. $M^{-1} = \begin{pmatrix} 1 & 3 \\ 2 & 0 \end{pmatrix}$

E. M has no inverse

Question 40. If $E = \frac{-i\sqrt{a-3}-2}{a+1}$, $a > 3$, which of the following is equal to $\frac{1}{E} + 2$?

A. $i - \frac{1}{4}$

B. $i\sqrt{a-3}$

C. $3 + a$

D. $i\sqrt{a+3}$

E. ∞